The Line Intersect Method In Forest Fuel Sampling

BY
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Abstract. A method for estimating wood volume on the ground is described. It requires only a diameter tally of pieces intersected by a sample line, and application of a simple formula. Theory for the formula is presented, and practical application discussed. The effect of bias in orientation of wood pieces can be largely overcome by running sample lines in two or more directions. The method was demonstrated indoors with match splints scattered on a 54-inch square and tested on a 20-acre cutover area. It has potential value for measuring fuel quantities in fire research.

IN FOREST FIRE RESEARCH it is often necessary to know the quantity of slash or other fuel present on the ground. Fine fuel is most easily measured by the collection and weighing of samples, but the estimation of larger-sized fuel would be simplified by a method requiring only a tally of the pieces intersected by a sample line. Such a line intersect technique was first successfully applied by Warren and Olsen[1] in measuring the volume of post-logging residue above a certain size limit in *Pinus radiata* plantations in New Zealand. Their method, developed for a special situation, produced volume estimates of good precision at much lower cost than could be achieved by conventional area sampling but required a preliminary test for bias in the orientation of the pieces. The method described here differs from that of Warren and Olsen in being generally applicable to the estimation of wood volume on the ground in any form, and in not requiring a preliminary test for orientation bias. The essence of the theory is the same as originally stated by Warren and Olsen, but an alternate derivation is presented that leads directly to the basic equation for the case of random orientation.

The tallying rules for the method are given, and its limitations and sources of error are discussed. Two tests were carried out, one in the laboratory and the other in the field.

Rules and Formulae

Tallying rules.

1. Lay a line of known length across the area to be studied.
2. Record the diameter of every piece of wood intersected.
3. If the sample line crosses the end of a piece, tally only if the central axis is crossed.
4. If the sample line passes exactly through the end of a piece's central axis, tally every second such piece.
5. Ignore any piece whose central axis coincides with the sample line.
6. If the sample line crosses a curved piece more than once, tally each crossing.

Rules 4, 5, and 6 are obviously of slight practical importance, but are included to cover all possibilities. Piece length and crossing angle need not be recorded.

Formulae.

The basic formula, when all factors are in the same units, is

$$V = \frac{\pi^2 \Sigma d^2}{8L}$$

where

V is volume of wood per unit area

d is piece diameter

L is length of sample line

If weight is desired, the volume estimate is simply multiplied by the specific gravity of the wood. That is,

$$W = \frac{\pi^2 S \Sigma d^2}{8L}$$

where

W is weight per unit area

S is specific gravity

These formulas can be modified to fit any set of units. For example, if d is recorded in inches, L measured in feet, and the answer desired in tons per acre, then

$$W = \frac{11.65 S \Sigma d^2}{L}$$

The basic formula depends on three assumptions:

1. The pieces are cylindrical. However, the presence of taper probably introduces no error.
2. All pieces are horizontal. However, the vertical angle can be quite large before the error is serious.
3. The pieces are randomly oriented. Bias in orientation can be corrected by special factors determined in field trials (the approach of Warren and Olsen), or by running sample lines in two or more directions and applying the basic formula.

All these limitations are discussed in more detail later.

Theory

Suppose that a sample line of length L crosses an area containing many horizontal cylinders of various lengths, diameters, and orientations. The sample line will cross cylinders at various angles, making a series of vertical elliptical cross sections that, if summed, would provide the required volume estimate. To visualize this, imagine the line to have infinitesimal width, which then cancels out in the division of total volume by total sampled ground area. The volume per unit area can thus be stated in terms of cross-sectional area per unit length of line; the net dimension, length, is the same in both expressions. Suppose that all possible orientations are equally represented throughout the sampled area and that the number of intersections is statistically large. Then, no matter what its actual crossing angle, each intersection can be shown to have an expected cross-sectional area that is the sum of all possible elliptical areas each weighted by its fractional probability, and that depends only on the piece diameter.

Consider a particular cylinder, intersected at angle u as shown in Figure 1 and with elliptical cross-sectional area x. Area x has its minimum value (a circle) when angle $u = \pi/2$ and tends to infinity as u approaches zero. However, as angle u decreases, the probability of intersection decreases, dependent on the piece's effective length perpendicular to the sample line; the ratio of this effective length to the actual length is sin u (in Fig. 1, the ratio of b to c). That is, the probability of intersection is proportional to sin u. It then follows from probability theory that the probability of u having a given value once intersection has

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[1] Warren, W. G., and P. F. Olsen. A line intersect technique for assessing logging waste. For. Sci. 10:267–276. 1964.

occurred is also proportional to sin u. Since angle u and area x are both continuous variables, probability density is used for probability and the expected area is obtained by integration. The probability density of u after intersection is then $p_x = k \sin u$, where k is a proportionality constant. Since intersection has occurred at some definite angle u, the sum of the probabilities must equal 1; i.e.,

$$\int_0^{\pi/2} p_x \, du = \int_0^{\pi/2} k \sin u \, du = 1$$

The integral of sin u from zero to $\pi/2$ is 1; therefore $k = 1$ and the probability density of u is simply sin u.

Now the area x of the elliptical intersection as a function of angle u (Fig. 1) is given by

$$x = \frac{\pi d a}{4} = \frac{\pi d^2 \operatorname{cosec} u}{4}$$

The expected area E_x is then

$$E_x = \int_0^{\pi/2} x \cdot p_x \, du$$

$$= \int_0^{\pi/2} \frac{\pi d^2 \operatorname{cosec} u}{4} \cdot \sin u \, du$$

$$= \frac{\pi d^2}{4} \int_0^{\pi/2} du = \frac{\pi d^2}{4} \cdot \frac{\pi}{2}$$

$$= \frac{\pi^2 d^2}{8}$$

Each crossing can thus be assumed to occur at the angle whose sine is $2/\pi$ (i.e., 39½°), and its expected cross-sectional area is just $\pi/2$ as great as would result from a crossing at right angles to the sample line.

The required estimate of wood volume per unit area V along the sample line is the sum of the expected areas of the various intersections, divided by the length of line L. That is,

$$V = \frac{\pi^2 \Sigma d^2}{8L}$$

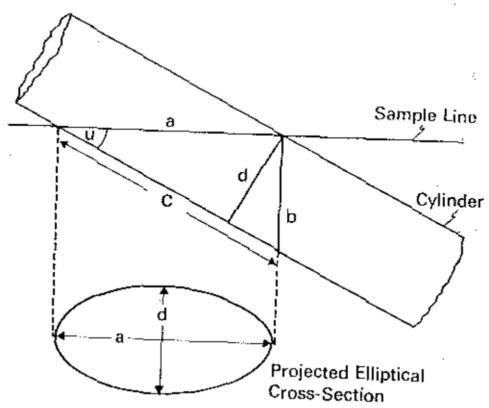

FIGURE 1. *The elliptical cross section at the intersection of sample line and cylinder.*

The result is independent of individual piece length and also of the size of the sampled area.

The above proof requires a complete elliptical cross section for each tallied intersection. But, when the sample line intersects one or both ends of a piece, there is a deficit, since the ellipse is truncated at one or both ends. According to Rule 3, however, only when its central axis is crossed is a piece tallied; the untallied bits of cross section from intersections that do not cut the central axis can then be shown to make up the deficit.

As angle u approaches zero in Figure 1 the limiting case is the perfect coincidence of sample line with central axis. Since the sample line cannot cross the central axis when angle u equals zero, such a piece should be ignored; in practice it need rarely be considered since its probability is zero.

Sources of Error in the Line Intersect Method

Provided the basic assumptions are met, the method gives an unbiased estimate of wood volume along the sample line. The theory, however, applies strictly to randomly oriented cylinders lying on a horizontal surface. Listed below are some ways in which field conditions fall short of the ideal model, and the nature of the resulting errors.

Taper. The volume of a tapered piece is the sum of all its right circular cross sections of infinitesimal thickness. Since the measurement of any one diameter should be an unbiased sample of the piece, the presence of taper consequently ought not to affect the volume estimate.

Axial asymmetry. If some pieces are not symmetrical about their central axes, it is reasonable to suppose that, on the average, representative diameters will be tallied.

Tilt. If the pieces were tilted at an angle h, their chances of being crossed would be reduced by the factor $1 - \cos h$, and the volume estimate would be low

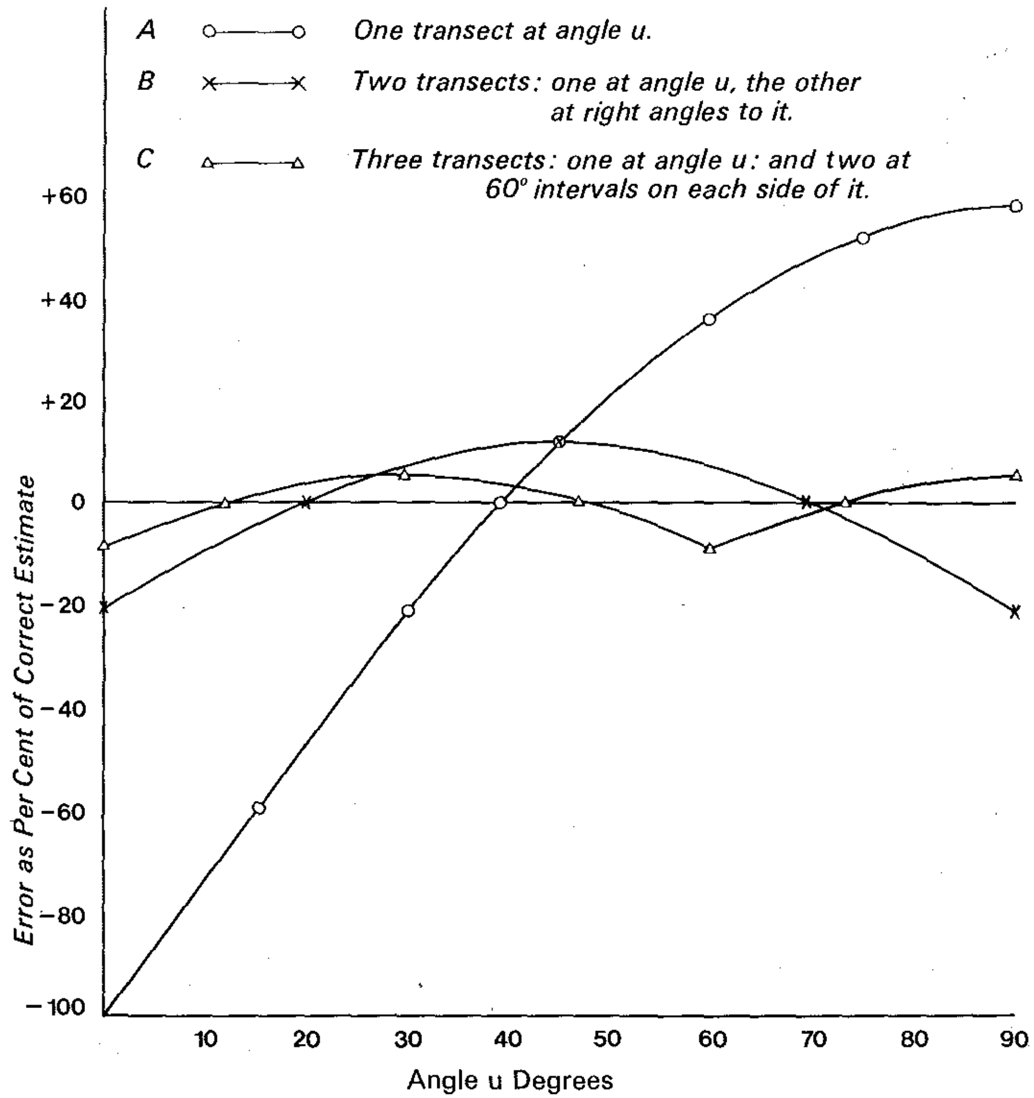

FIGURE 2. *Extreme possible errors in the line intersect method for one, two and three sample lines at uniform angular intervals.*

by that amount. However, this error is considerable only at fairly high tilt angles. For example, it is only 0.4 percent at 5 degrees, and is still less than 10 percent at 25 degrees. Minor tilt can thus be safely ignored.

Sloped surface. When the sampled area lies on a slope, the volume V per unit area can readily be converted to the horizontal basis. Multiply V by the ratio of slope area to equivalent horizontal area.

Curvature. The sample line may occasionally cross a crooked piece twice. The simple rule of tallying each intersection may not be quite correct mathematically, but is close enough for practical purposes.

Orientation bias. If the pieces are not randomly oriented, the effect of the bias can be greatly reduced by running sample lines in more than one direction and averaging the results. Since no angles are measured while applying the method, the error due to orientation bias cannot be calculated; some way of estimating its limits is therefore desirable. The maximum possible error would occur if all pieces were aligned in the same direction. Figure 2 shows the error for this extreme case (a) when one sample line is run at angle u, (b) when another sample line is added at right angles to the first, and (c) when two sample lines are added at 60° intervals on each side of the first. For the single sample line, the function is $\frac{\pi}{2}\sin u$, which gives the correct estimate when u is 39½°, the angle whose sine is $2/\pi$. The error ranges in Figure 2 are (a) from -100 to $+57$ percent with one sample line, (b) from -21 to $+11$ percent with two lines at right angles, and (c) from -9 to $+5$ percent with three lines at 60°. Obviously, even with an obvious orientation bias, three lines at 60° is a fairly safe condition regardless of how they are placed. If the direction of the bias is constant, error can be minimized by a choice of angle u; if it varies within the sample area, then angle u had best be chosen at random.

The principal remaining question is: what total length of line is necessary to provide an estimate with a chosen degree of reliability? A good analogy is the familiar mensurational problem of how much strip cruising is necessary to define the stand table of a forest tract. It is therefore possible to calculate the reliability of the line intersect method by dividing the total sample line into sections and testing the mean volume estimate per section.

Testing the Method

The line intersect theory was demonstrated in the laboratory and tried in the field. The laboratory demonstration consisted in scattering 2-inch-long match splints on a 54-inch square and comparing the true number present with an estimate obtained by the line intersect method. Within the square a grid of 22 lines was drawn, 11 lines one way and 11 at right angles to them. The crossings on each line were tallied and the number of matches was estimated with the following formula, readily derived from the basic one:

$$N = \frac{\pi n A}{2 L y}$$

where

N is total number of matches on square

n is total number of crossings

A is test area

L is total length of sample line

y is length of each match

(This formula can be used to estimate the number of equal-length pieces on an area of any shape.)

In all, 20 tests were performed with numbers of matches ranging from 50 to 1,077. The true and estimated numbers are compared in Table 1. The standard error of each estimate was determined from the number of crossings per line on the 22 sample lines. Only one estimate was outside the 95 percent confidence limit, as might indeed be expected in a series of 20 tests. Table 2 also shows that the standard errors increased in absolute value as the number of matches increased, but decreased when expressed as a percent of the true number. These results show that the ideal conditions of the mathematical theory can be fairly well reproduced in the laboratory.

For the field trial a 20-acre area was chosen on which all trees had been felled, topped, and bucked into 16-foot lengths. There was no obvious bias in the orientation of the pieces. The original stand was a mixture of hardwoods and conifers with diameters averaging about 6 inches and running to a maximum of 15 inches. Nineteen 100-foot sections of sample line were run in three different directions, and all crossings over 1.5 inches in diameter were tallied in 1-inch classes. Each of the 680 crossings was measured with a caliper, and two men completed the job in 5 hours with the results shown in Table 2. The field trial thus fulfilled its purpose, which was to show

TABLE 1. Estimates of the number of match splints on a 54-inch square by the line intersect method.

True number of matches	Estimated number ± standard errors	Deviation as percent of true number	Average number crossings per sample line
50	57 ± 10	14	1.3
50	49 ± 10	− 2.0	1.2
70	69 ± 10	− 1.4	1.6
76	83 ± 11	9.2	2.0
95	114 ± 16	20	2.7
101	105 ± 14	4.0	2.5
113	85[1] ± 12	−25	2.0
114	111 ± 16	− 2.6	2.6
131	164 ± 24	25	3.9
226	210 ± 18	− 7.1	5.0
341	380 ± 31	11	9.0
412	384 ± 30	− 6.8	9.1
473	472 ± 32	− 0.2	11.1
502	534 ± 50	6.4	12.6
537	521 ± 45	− 3.0	12.3
596	639 ± 41	7.2	15.1
699	743 ± 43	6.3	17.5
809	886 ± 51	9.5	20.9
915	920 ± 44	0.5	21.7
1,077	1,100 ± 84	2.1	25.9

[1]Outside 95 percent confidence limit.

TABLE 2. Field trial of the line intersect method.

Number of 100-ft sample lines	19
Average number of crossings per line	36
Average volume estimate	2,701 ft³/A
Standard error	±141 ft³/A (5.2% of mean)
95% confidence limit	±296 ft³/A (11.0% of mean)

that the line intersect method could provide an estimate of acceptable reliability in reasonable time.

For two reasons determination of the true volume per acre was not attempted. First, the basic validity of the method was regarded as proved. Second, volume measurement by any other method would obviously have been a formidable, much lengthier task with no more guarantee of freedom from bias than the line intersect result itself. This experience matches Warren and Olsen's account of a test in which 4 hours of line intersect sampling yielded a better estimate than did 20 hours of conventional area sampling. Without a practical way of verifying line intersect field results, there seems then to be no reason for not accepting them as correct, subject to the sources of error discussed in the previous section.

The optimum lower diameter limit for the line intersect method is probably between 0.5 and 1.5 inches. Very small sizes are more efficiently sampled by weighing material collected on narrow strips or small plots.

It is apparent from the number of crossings per section of line that the field trial represents a denser concentration of material than any of the laboratory match tests. Probably the length of one unit of sample line is best chosen to ensure a fair number of crossings per section, say 20 or more; the number of sections required for the chosen level of reliability will then depend on the amount of variation in the diameter and spacing of the pieces.

Statistical Methods

By George W. Snedecor and William G. Cochran. 1967. The Iowa State University Press, Ames, Iowa. pp. 593.

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Comparative Seedling Growth Of Four Hardwood Species

BY
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Abstract. Seedlings of *Populus balsamifera* L., *Liriodendron tulipifera* L., *Ulmus americana* L., and *Acer rubrum* L. were grown in a fiberglass greenhouse. Total dry weight after 20 weeks was, (a) proportional to the percentage of dry matter allocated to leaf production, (b) relatively little affected by differences in seed size and net assimilation rates and (c) inversely related to the shade tolerance of the species involved.

GROWTH, expressed as dry matter accumulation, depends on the efficiency of utilization of solar energy by the tree, or net assimilation rate, and on the size of the assimilating system, or leaf area. Heath and Gregory (1938), Nichiporovich (1956) and Watson (1956) have all concluded that differences in yields among agricultural crops are largely a result of differences in leaf area development, differences in net assimilation rates being relatively small. Matthews (1963) in a paper on the physiological bases for differences in productivity of forest trees came to a similar conclusion but experimental data appear lacking.

The objective of this experiment was to see how differences in growth among four tree species of varying shade tolerance were related to variations in net assimilation rate and leaf area development.

Procedures

The species used in the experiment and their shade tolerances were:

1. Very intolerant—balsam poplar (*Populus balsamifera* L.)
2. Intolerant—yellow-poplar (*Liriodendron tulipifera* L.)
3. Intermediate—American elm (*Ulmus americana* L.)
4. Tolerant—red maple (*Acer rubrum* L.)

These species were chosen because of the range of shade tolerance as indicated by Baker (1949), the availability of the seed, and their simplicity as experimental material.

In mid-May locally collected seed was sown in approximately one-gallon cans in a mixture of clay loam, humus, and sand. Holes in the bottom of the cans insured good drainage. The cans of each species were placed at random in each of five blocks in a fiberglass greenhouse. The plants were watered daily to approximately field capacity. A water soluble fertilizer was added twice during the growing season in order to maintain soil fertility.

At 2, 4, 8, 12, 16, and 20 weeks after germination, two plants of each species were randomly selected from each block. At the time of sampling, height, diameter and oven dry weight of stems, roots, and leaves were measured. Leaf area was estimated by weighing filter paper cut to the shape of the leaves.

Net assimilation rates (*NAR*) were determined using the formula of Briggs *et al.* (1920) for the three periods 8–12 weeks, 12–16 weeks and 16–20 weeks:

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