Cubic-Foot Volume of Loblolly Pine to Any Merchantable Top Limit

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ABSTRACT. Foresters commonly wish to predict tree volume for various top diameters. However, tree volume equations are generally restricted to specific top limits. Further, volume equations for various top limits often cross illogically. This study was conducted with the objective of developing logically related cubic-foot volume estimates for any desired top diameter limit. The approach taken was to predict total stem volume and to convert total volume to merchantable volumes by applying predicted ratios. Results reported here enable users to employ relatively simple equations to obtain cubic-foot volume to any top diameter limit. Volume between any two specified diameters on the stem can be obtained by subtraction.

Foresters use standard volume equations to estimate the volume of standing trees through the measurement of diameter at breast height (dbh) and total or merchantable height. These volume equations commonly provide estimates of the contents of tree boles (in some specified units) from stump height to a fixed top diameter. However, merchantability standards change rapidly and it is often desirable to obtain volume estimates in given units but to various top diameters. The usual approach taken to solve the problem of multiple top limits has been to predict the volume to various top diameters as a function of tree dbh and total height values. Unconstrained, independent equations to several top limits may have the undesirable characteristic of illogically crossing within the range of observed data (that is, at given combinations of dbh and total height the predicted volume to a 4-inch top, for example, may exceed that for a 3-inch top). One means of eliminating this difficulty is to constrain the volume equations to various top limits to differ by a constant amount (Bailey and Clutter 1970). This may be a satisfactory solution for small top diameters where one would expect the merchantable volume lines to be nearly parallel to the total volume line. However, as top diameters become more restrictive the slope of the merchantable volume line should become steeper without crossing volume lines for smaller merchantable top diameter limits.

The objective of this investigation was to provide estimates of cubic-foot volume to any desired top diameter limit while insuring that the predicted volumes were logically related. The approach taken was to predict the total stem volume and the ratio of merchantable stem volume to total stem volume for any given top diameter. Thus, with two relatively simple equations, foresters can obtain volume to any top diameter limit or the volume between any two specified diameters on the stem by subtraction. Although the specific equations presented are for cubic-foot volume, the general approach should be applicable to other volume units and to weight prediction.

DATA

Data available for this study were from trees felled on temporary yield plots in plantations and natural stands of loblolly pine (see Burkhart *et al.* 1972a and 1972b for a complete description of the yield plots). Plantation-grown sample trees were from the Piedmont and Coastal Plain regions of Virginia and from the Coastal Plain regions of Delaware, Maryland, and North Carolina. Sample tree data from natural stands were obtained in the Piedmont and Coastal Plain regions of Virginia and in the Coastal Plain of North Carolina.

On each plot, two single-stemmed trees were felled and cut into 4-foot sections. In addition to measuring the tree dbh and total height, diameters inside and outside bark were measured at the stump and at 4-foot intervals up the stem to an approximate 2-inch top diameter, outside bark. Although stump heights were not measured, all were approximately 0.5 feet. There was a total of 472 sample trees from plantations and 232 from natural stands. The sample trees from plantations ranged from 3 to 12 inches in dbh and from 30 to 90 feet in total height; those from natural stands spanned from 5 to 14 inches dbh and from 30 to 90 feet in total height.

Cubic-foot volumes inside bark (ib) and outside bark (ob) were calculated for ob top diameters of 2, 3, 4, . . . *etc.* inches until top diameter was equal to or exceeded dbh. At each ob top diameter point, the diameter ib was also computed. Volume above the last stem measurement was computed by treating the segment as a cone. The cone volume was added to the total volume computed for the 4-foot segments to obtain total stem volume. For each top limit, the ratio of merchantable volume (ob) to total stem volume (ob) was computed; identical computations were carried out for ib volumes.

TOTAL STEM VOLUME EQUATIONS

A search for the best total stem cubic-foot volume prediction equation was conducted. From the total of 472 plantation-grown trees, 100 were selected at random and withdrawn for evaluation purposes. The remaining 372 observations were used to estimate the parameters in three total stem volume (ob and ib) models chosen for comparison. Models compared were:

$$V = \beta_{01} + \beta_{11}D^2H \quad (1)$$

$$V = \beta_{02} + \beta_{12}D^{\beta_{22}}H^{\beta_{32}} \quad (2)$$

$$V = D^2/(\beta_{03} + \beta_{13}/H) \quad (3)$$

where

V = total stem cubic-foot volume (ob or ib)
D = tree dbh in inches
H = total tree height in feet
β_{ij} = coefficients to be estimated from sample data

Model (1) is the well-known combined-variable function (Spurr 1952); Model (2) is a modification of the combined-variable function in which the exponents of D and H are estimated from the sample data; and Model (3) is a total cubic-foot volume function proposed by Honer (1965).

The resulting equations were evaluated by predicting the volume for each of the 100 trees withheld and subtracting the predicted volume from the observed volume. Analysis of differences between observed and predicted volumes showed that the simple combined-variable function [Model (1)] performed as well as, if not better than, any of the alternatives evaluated.

Inclusion of an expression for tree form may improve the precision of predictions, but the measurement of form is difficult in many field situations. Consequently, stand characteristics (age, site index, number of trees per acre, and basal area per acre) which might serve as surrogates for form were included with D^2H to see if some predictive improvement could be realized. Analysis of residuals for the data set of 100 trees that were withheld showed no improvement for models which included stand variables as well as D^2H. Consequently, the combined-variable function [Model (1)] was adopted for predicting total cubic-foot volume for plantation-grown loblolly pine.

Similar analyses to those performed for plantation-grown sample trees were carried out for the sample trees from natural stands. The results indicated that the combined-variable function performed at least as well as the other alternatives evaluated and that the inclusion of stand characteristics with D^2H did not improve predictive ability. Coefficients for the combined-variable model were estimated from the total data set of 472 plantation-grown trees and from the total of 232 natural stand-grown trees. The coefficients are presented in Table 1.

TABLE 1. Coefficients for estimating total cubic-foot volume, outside and inside bark, of loblolly pine trees.

	b_0[1]	b_1[1]
	Plantation-grown trees	
Outside bark	0.34864	0.00232
Inside bark	0.11691	0.00185
	Natural stand-grown trees	
Outside bark	0.27611	0.00253
Inside bark	0.00828	0.00205

[1] *Equation: $V = b_0 + b_1D^2H$ where V = cubic-foot volume (outside or inside bark), D = dbh in inches, and H = total height in feet.*

VOLUME RATIO EQUATIONS

After establishing equations to predict total stem volume, models for predicting the ratio of merchantable stem volume divided by total stem volume were evaluated. All models evaluated were conditioned so that when diameter at the top equalled zero the ratio equalled one. The addition of total tree height did not significantly reduce the residual sum of squares for any of the models included, and the model which gave the most satisfactory results was a nonlinear model of the form:

$$R = 1 + \beta_1(D_t^{\beta_2}/D^{\beta_3}) \quad (4)$$

where

D = dbh in inches
D_t = top diameter (outside or inside bark) in inches
R = merchantable cubic-foot volume (outside or inside bark) to top diameter D_t/total stem volume (outside or inside bark) in cubic feet
β_i = constants to be estimated from the data

Using nonlinear regression techniques, parameters of the ratio model were estimated for plantation-grown and for natural stand-grown trees. These parameter estimates are displayed in Table 2. Plotting the resultant volume predictions showed that for small top diameters the

TABLE 2. Coefficients for estimating the ratio of merchantable stem volume divided by total stem volume for loblolly pine trees.

Ratio (R)	Coefficients[1] b_1	b_2	b_3
	Plantation-grown trees		
	Top diameter (D_t) outside bark		
ob	−0.32354	3.1579	2.7115
	Top diameter (D_t) inside bark		
ob	−0.77135	3.3736	3.1080
	Top diameter (D_t) outside bark		
ib	−0.35206	3.0763	2.6540
	Top diameter (D_t) inside bark		
ib	−0.82347	3.2822	3.0388
	Natural stand-grown trees		
	Top diameter (D_t) outside bark		
ob	−0.42919	3.4407	3.1090
	Top diameter (D_t) inside bark		
ob	−0.91416	3.7221	3.4992
	Top diameter (D_t) outside bark		
ib	−0.48402	3.3835	3.0881
	Top diameter (D_t) inside bark		
ib	−1.0117	3.6431	3.4545

[1] *Equation: $R = 1 + b_1(D_t^{b_2}/D^{b_3})$ where D = dbh in inches, D_t = top diameter (outside or inside bark) in inches, R = merchantable cubic-foot volume (outside or inside bark) to top diameter D_t/ total stem volume (outside or inside bark) in cubic feet.*

volume lines were nearly parallel to the total volume line, while the slopes became increasingly steeper (but without illogical crossing) as top diameter became larger. Thus, the ratio equation gave logical and consistent results when converting total stem volume to merchantable volume at any desired top limit.

APPLYING THE EQUATIONS

The outside bark ratios (Table 2) can be used to convert the total stem volumes (ob) predicted from the appropriate equation in Table 1 to merchantable volumes to various top diameters outside or inside bark. Likewise, inside bark ratio equations in Table 2 can be applied with the appropriate total stem volume (ib) equation in Table 1 to obtain merchantable volumes, inside bark, to any desired top diameter ob or ib.

To illustrate the application of the prediction equations, suppose that merchantable volumes (ob) to 3- and 4-inch tops (ib) are desired for a plantation-grown loblolly pine tree that measures 8 inches dbh and 50 feet total height. One first selects the appropriate total volume equation (ob) from Table 1 and substitutes the tree dimensions:

$$V = 0.34864 + 0.00232(8)^2(50) = 7.77 \text{ cu. ft.}$$

Cubic-foot volume (ob) to a 3-inch top diameter (ib) is computed by substituting into the appropriate ratio equation from Table 2 and multiplying the ratio times the total volume, outside bark. Substituting the selected top limit and the measured dbh yields:

$$R = 1 - 0.77135(3^{3.3736}/8^{3.1080}) = 0.951$$

which is multiplied times the total volume (ob) to compute cubic-foot volume (ob) to a 3-inch top (ib):

$$V_3 = (7.77)(0.951) = 7.39 \text{ cu. ft.}$$

Analogously, cubic-foot volume to a 4-inch top is computed as:

$$R = 1 - 0.77135(4^{3.3736}/8^{3.1080}) = 0.871$$

$$V_4 = (7.77)(0.871) = 6.77 \text{ cu. ft.}$$

Cubic-foot volume, outside bark, between top limits of 3- and 4-inches, inside bark, can, of course, be computed by subtraction (7.39 − 6.77 = 0.62 cu. ft.).

The methods and equations presented in this paper will hopefully be of value to foresters who need to estimate contents of trees to various top diameter limits. Situations in which more detailed tree volume or weight information is desired will naturally require more complex analytical techniques than those employed here.

Literature Cited

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