Cubic-Foot Volume of Loblolly Pine to Any Height Limit

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*ABSTRACT. Flexible methods for computing the contents of various portions of tree boles are necessary with today's changing utilization standards. Equations are presented for estimating the cubic-foot volume of loblolly pine (*Pinus taeda *L.) trees to any desired height. The procedure involves predicting total stem volume and converting total volume to merchantable volume.*

Utilization standards are changing rapidly, thus necessitating methods for estimating the contents of various portions of tree boles. Diameter is the most common limiting factor for utilization, and equations for estimating the cubic-foot volume of loblolly pine to any merchantable top diameter have been published by Burkhart (1977). However, products are sometimes defined in fixed-length multiples. Consequently equations are presented here for estimating the volume of any specified length of bole.

DATA

Sample data used in this study were from trees felled on plots in loblolly pine plantations and natural stands. Plots of 0.1-acre were randomly located in plantations and natural stands selected for sampling in a growth and yield study. On each plot two trees (the tenth and twentieth trees tallied by d.b.h.) were felled for detailed measurements. Sample tree data from plantations were obtained in the Virginia Piedmont and Coastal Plain and the Coastal Plain of Delaware, Maryland, and North Carolina. Sample trees from natural stands were from the Virginia Piedmont and Coastal Plain and the Coastal plain of North Carolina. Table 1 shows the distribution of sample trees by d.b.h. and height classes for plantations and natural stands.

These trees were cut into 4-foot sections. Stump heights were not measured but a constant stump height of 0.5 foot was assumed. Tree d.b.h. and total height were measured to the nearest 0.1 inch and 0.1 foot, respectively. Diameters outside bark (ob) and inside bark (ib) at the stump and at 4-foot intervals up the stem to an approximate 2-inch top diameter ob were also recorded. Cubic-foot volumes ob and ib were determined by using Smalian's formula for each 4-foot section. Merchantable volumes were obtained by summing the volumes of the sections to the appropriate limits (each measurement was considered as a top-height limit). Total stem volume was obtained by adding the top volume to the merchantable volume which has the highest top limit, assuming that the top was a cone.

EQUATION FOR TOTAL CUBIC-FOOT VOLUME

After evaluating a number of models Burkhart (1977) concluded that the simple combined variable equation performed as well or better than any of the alternatives compared. The combined variable equation is of the form:

$$V = b_0 + b_1 D^2 H \tag{1}$$

where

- V = total cubic-foot volume (ob or ib) above the stump,
- D = tree d.b.h. in inches,
- H = total tree height (from the ground) in feet,
- b_0, b_1 = coefficients to be estimated from the data.

Statistical tests revealed that, for these data, it would be possible to combine information from the Piedmont and Coastal Plain physiographic provinces, but that separate equations would be needed for plantations and natural stands.

Table 2 shows the coefficients for predicting total cubic-foot volume ob and ib for both natural stand-grown and plantation-grown trees. The coefficients in Table 2, which are the same as those presented by Burkhart (1977), are shown here for completeness.

Equations for predicting total cubic-foot volume account for at least 97 percent of the variation in observed volume (i.e. r^2 for each equation was 97 percent or more).

Table 1. Distribution of sample trees by total height and diameter at breast height.

Diameter[1] at breast height (inches)	Total height (feet)[1]								
	20	30	40	50	60	70	80	90	Total
Plantations									
2	2								2
3	12	5		1					18
4	23	32	7						62
5	7	42	38	12	1				100
6	1	22	47	37	3				110
7		5	16	33	8				62
8			5	21	13	3			42
9			1	10	4	2			17
10				1	6		1		8
11				1	3		1		5
12							1		1
Total	45	106	114	116	38	5	3		427
Natural stands									
5		1	11	6	2	1			21
6			4	12	7				23
7			6	21	17	2			46
8			1	13	16	6			36
9				5	17	7	1		30
10				3	9	11	3	1	27
11				1	5	6	1	1	14
12						3			3
13						3	1	3	7
14						1		1	2
Total		1	22	61	73	40	6	6	209

[1]Diameter and height classes are midpoint; e.g. 10-inch diameter class includes 9.6-10.5 and 50-foot height class includes 45.1-55.0.

VOLUME RATIO MODEL

Several models for predicting the ratio of merchantable volume to total volume were evaluated. The model which gave the most satisfactory results in accuracy and precision is a nonlinear model of the form:

$$R = 1 + b_1 (p^{b_2}/H^{b_3}) \qquad (2)$$

where

- p = distance in feet from the tree tip to the limit of utilization,
- H = total tree height (from the ground) in feet,
- R = v/V,
- v = merchantable cubic-foot volume (ob or ib) from the stump to the utilization limit, specified by p,
- V = total cubic-foot volume (ob or ib) above the stump,
- b_i = regression coefficients to be estimated from the sample data,
- i = 1, 2, 3

Equation (2) was conditioned so that R is 1 when p equals 0. This model was chosen due to its satisfactory performance, its simple form, its ease of application, and also because only three parameters need to be estimated from the sample data.

The same grouping was used here as that in predicting total volume; that is, Piedmont and Coastal Plain data were pooled together whereas natural-stand and plantation trees were held separate. Nonlinear regression techniques (modified Gauss-Newton method) were used to compute parameter estimates, which are shown in Table 3 for ob and ib volume ratios. The equations for predicting volume ratios account for 99 percent of the variation (*i.e.*, the ratio of regression sum of squares to total sum of squares was 99 percent for each equation).

Table 2. Regression coefficients for estimating total cubic-foot volumes outside and inside bark of loblolly pines from different sources.

Source	Coefficients[1]	
	b_0	b_1
Plantation-grown trees		
Outside bark	0.34864	0.00232
Inside bark	0.11691	0.00185
Natural stand-grown trees		
Outside bark	0.27611	0.00253
Inside bark	0.00828	0.00205

[1]Equation: $V = b_0 + b_1 D^2H$, where V = total cubic-foot volume, D = d.b.h. in inches, and H = total height in feet.

APPLICATIONS

The following example serves to illustrate the use of this model. Consider a natural stand-grown loblolly pine tree measuring 10 inches d.b.h. and 60 feet total height and suppose that merchantable volumes ib to heights of 40 and 50 feet are desired for this tree. The total volume ib is first computed

Table 3. Coefficients for estimating the ratio of merchantable cubic-foot volume to total cubic-foot volume, outside and inside bark, for loblolly pines from different sources.

Source	Coefficients[1]		
	b_1	b_2	b_3
Plantation-grown trees			
Outside bark	−0.706413	2.475627	2.392790
Inside bark	−0.731856	2.399442	2.319504
Natural stand-grown trees			
Outside bark	−0.564848	2.272322	2.143705
Inside bark	−0.642258	2.203533	2.099241

[1]Equation: $R = 1 + b_1 (p^{b_2}/H^{b_3})$, where p = distance in feet from the tree tip to the utilization limit, H = total height in feet, R = ratio of merchantable cubic-foot volume to total cubic-foot volume.

from Equation (1), using the appropriate coefficients from Table 2:

$$V = 0.00828 + 0.00205\ (10)^2\ (60) = 12.31 \text{ cu. ft.}$$

The distance from the tree tip, p, is 60 − 40 = 20 feet and total height, H, is 60 feet. Substituting into Equation (2) gives:

$$R = 1 - 0.642258\ (20^{2.203533}/60^{2.099241}) = 0.9125$$

which is multiplied by the total volume ib, V, to yield cubic-foot volume ib to 40 feet high:

$$V_{40} = (0.9125)\ (12.31) = 11.23 \text{ cu. ft.}$$

Volume ib to a height of 50 feet is computed using the same method with p = 10 feet:

$$R = 1 - 0.642258\ (10^{2.203533}/60^{2.099241}) = 0.9810$$

$$V_{50} = (0.9810)\ (12.31) = 12.08 \text{ cu. ft.}$$

Cubic-foot volume ib of the bole segment between 40 and 50 feet high is:

$$12.08 - 11.23 = 0.85 \text{ cu. ft.}$$

Volume to any specified top diameter can be calculated from the model presented by Burkhart (1977). Therefore, volume of a bole section having a fixed diameter at one end and a specified height at the other end can be easily computed by subtraction.

In some cases, products are defined in bolts of a fixed length where the small end of each bolt must have a diameter greater than a specified value. To compute small-end diameters, one can calculate the volume ratio for the length of interest, substitute this ratio into the appropriate volume ratio equation in Burkhart (1977) (based on top diameter and d.b.h.), and solve for the diameter at the top of the segment. The volume ratio equations in Burkhart (1977) are of the form

$$R = 1 + b_1(D_t^{b_2}/D^{b_3})$$

where

D = d.b.h. in inches
D_t = top diameter in inches
R = merchantable cubic-foot volume to top diameter D_t/total stem volume in cubic feet

Thus, if R and D are specified, one can solve for top diameter as:

$$D_t = \left[\frac{(R-1)D^{b_3}}{b_1}\right]^{1/b_2}$$

Following through with the numeric example in this paper, suppose that the top diameter (ib) of the 40-foot segment is desired. The inside-bark volume ratio for that segment (0.9125), d.b.h. (10 inches), and appropriate coefficients from Burkhart (1977) are substituted into the above expression to give the top diameter:

$$D_t = \left[\frac{(0.9125 - 1)10^{3.4545}}{-1.0117}\right]^{1/3.6431} = 4.53 \text{ inches (inside bark)}$$

Through analogous procedures, the two ratio models can be used to solve for the height to a specified top diameter. Alternatively, taper equations, such as the one presented by Max and Burkhart (1976), can be employed to determine diameters at specified heights and heights to specified diameters. The Max and Burkhart (1976) taper equation is based on the same tree data as those used here.

A system composed of equations which can predict volume to any top diameter or height limit should be useful in forest inventory or yield estimation. Although the analyses were made only on the two available loblolly pine data sets, it is believed that such a system will perform reasonably well for other single-stemmed conifers.

Literature Cited

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